MATEMATİK II VİZE CEVAPLAR

① a) $\int (x+3)^2 x^5 dx = ?$

$= \int (x^2+6x+9)(x^5)\,dx$

$= \int (x^7+6x^6+9x^5)\,dx$

$= \frac{x^8}{8} + \frac{6}{7}x^7 + \frac{9}{6}x^6 + C$ //

b) $\int \frac{2^{-x}}{5+2^{-x}} dx = ?$

$5+2^{-x} = u$

$-2^{-x} dx = du$

$\int -\frac{1}{u}\,du = -\ln|u| + C$

$= -\ln|5+2^{-x}| + C$

c) $\int_1^2 \frac{2x-6}{(x+1)(2x-1)} = ?$

$\frac{2x-6}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{K}{2x-1}$

$= \int_1^2 \left(\frac{2}{x+1} - \frac{2}{2x-1}\right) dx$

$= (2\ln|x+1| - 2\ln|2x-1|)\Big|_1^2$

$= (\ln 3 - 2\ln 3) - (2\ln 2 - 2\ln 1)$

$= -2\ln 2$

② a) $\int_0^{\ln 2} \frac{e^{2x}}{1+e^{2x}} dx = ?$

$1+e^{2x} = u$

$2e^{2x} dx = du$

$= \int_0^{\ln 2} \frac{1}{u}\frac{du}{2} = \frac{1}{2}\int_0^{\ln 2} \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u|\Big|_0^{\ln 2}$

$= \frac{1}{2}\left(\ln|1+e^{2x}|\right)\Big|_0^{\ln 2}$

$= \frac{1}{2}\left[(\ln(1+e^{2\cdot\ln 2}) - (\ln(1+e^{2\cdot 0})\right]$

$= \frac{1}{2}\left[\ln(1+4) - (\ln 2)\right]$

$= \frac{1}{2}\left[\ln 5 - \ln 2\right] = \frac{1}{2}\ln 5/2$ //

b) $\int_{-2}^{0} |x+1|\,dx = ?$ $\quad |x+1| = \begin{cases} -(x+1) & \text{eğer } -2 \le x < -1 \\ (x+1) & -1 \le x \le 0 \end{cases}$

$= \int_{-2}^{-1} -(x+1)\,dx + \int_{-1}^{0} (x+1)\,dx$

$= \left(-\frac{x^2}{2} - x\Big|_{-2}^{-1}\right) + \left(\frac{x^2}{2} + x\right)\Big|_{-1}^{0}$

$= \left[\left(-\frac{1}{2}+1\right) - (-2+2)\right] + \left[0 - \left(\frac{1}{2}-1\right)\right]$

$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) = 1$ //

c) $\int_1^e \frac{1}{(1+\ln x)^2} \cdot \frac{dx}{x} = ?$ $\quad 1+\ln x = u$

$\frac{1}{x}dx = du$

$\int_1^e \frac{1}{u^2}\,du = -\frac{1}{u}\Big|_1^e = -\frac{1}{1+\ln x}\Big|_1^e$

$= \left(-\frac{1}{1+\ln e}\right) - \left(-\frac{1}{1+\ln 1}\right)$

$= \left(-\frac{1}{2}\right) - (-1) = 1/2$ //

③ a) $\int x(\ln x)^3\,dx = ?$

$(\ln x)^3 = u$ $\quad x\,dx = dv$

$3(\ln x)^2 \cdot \frac{1}{x}dx = du$ $\quad \frac{x^2}{2} = v$

$u\cdot v - \int v\,du$

$= (\ln x)^3 \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot 3(\ln x)^2 \cdot \frac{1}{x}\,dx$

$= \frac{x^2}{2}(\ln x)^3 - \int 3\frac{x^2}{2} \cdot \frac{1}{x}(\ln x)^2 dx$

$= \frac{x^2}{2}(\ln x)^3 - \frac{3}{2}\int x(\ln x)^2 dx$

$A = \int x(\ln x)^2 dx$

$(\ln x)^2 = u$ $\quad x\,dx = dv$

$2\ln x \cdot \frac{1}{x}dx = du$ $\quad \frac{x^2}{2} = v$

$u\cdot v - \int v\,du$

$(\ln x)^2 \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot 2\cdot\frac{1}{x}\cdot \ln x\,dx$

$= \frac{x^2}{2}(\ln x)^2 - \int x\ln x\,dx$

B

$B = \int x\ln x\,dx$

$\ln x = u$ $\quad x\,dx = dv$

$\frac{1}{x}dx = du$ $\quad \frac{x^2}{2} = v$

$u\cdot v - \int v\,du$

$= \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x}dx$

$= \frac{x^2}{2}\cdot\ln x - \frac{1}{2}\int x\,dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\frac{x^2}{2}$

A ve B yerine koyulursa sonuç bulunur.

(3) b) $\int 24x^2 e^{6x}\,dx = ?$

$u \cdot v - \int v \cdot du$

$24x^2 = u$ $\quad e^{6x}dx = dv$

$48x\,dx = du$ $\quad \frac{1}{6}e^{6x} = v$

$= 24x^2 \cdot \frac{1}{6}e^{6x} - \int \frac{1}{6}e^{6x} \cdot 48x\,dx$

$= 4x^2 e^{6x} - 8\int x e^{6x}\,dx$ $\quad A$

$A = \int x e^{6x}\,dx$

$u \cdot v - \int v\,du$

$x = u$ $\quad e^{6x}dx = dv$

$dx = du$ $\quad \frac{1}{6}e^{6x} = v$

$= x \cdot \frac{1}{6}e^{6x} - \int \frac{1}{6}e^{6x} \cdot dx$

$= \frac{x}{6}e^{6x} - \frac{1}{6} \cdot \frac{1}{6}e^{6x} = \frac{x}{6}e^{6x} - \frac{e^{6x}}{36} = A$

Cevap $\Rightarrow$ $4x^2 e^{6x} - 8\left[\frac{x}{6}e^{6x} - \frac{e^{6x}}{36}\right] + C$

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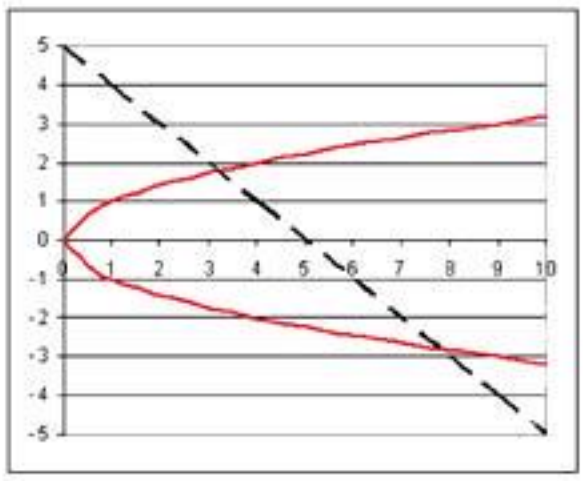
(4) a) $x = y^2$ $\quad y = -1$

$y = -x + 5$ $\quad y = 2$



$Alan = \int_{-1}^{2} (-y+5) - (y^2)\,dy$

$= \int_{-1}^{2} (-y^2 - y + 5)\,dy$

$= -\frac{y^3}{3} - \frac{y^2}{2} + 5y \Big/_{-1}^{2}$

$= \frac{16}{3} - \left(-\frac{31}{6}\right) = \frac{63}{6}\ br^2$ //

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b) $y = x^2 - 4$

$y = 3x$

$Alan = \int_{-1}^{4} (3x) - (x^2 - 4)\,dx$

$3x = x^2 - 4$

$x^2 - 3x - 4 = 0$

$1 \quad 1$

$x \quad -4$

$x \quad +1$

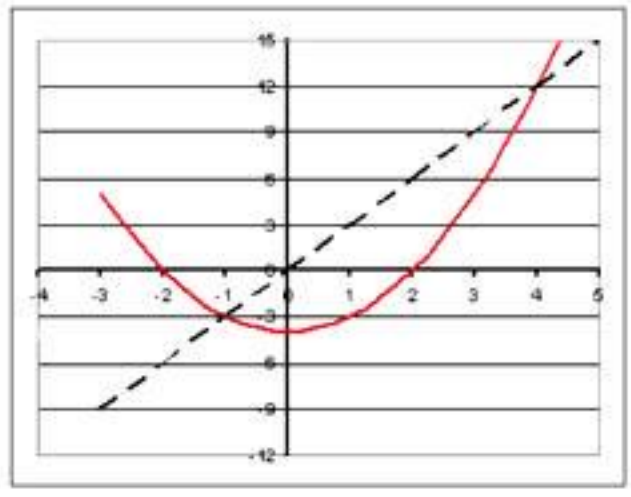
$x_1 = 4$

$x_2 = -1$

$= \frac{3}{2}x^2 - \frac{x^3}{3} + 4x \Big/_{-1}^{4}$

$= \left(24 - \frac{81}{3} + 16\right) - \left(\frac{3}{2} + \frac{1}{3} - 4\right)$

$= \frac{39}{3} - \frac{-13}{6} = \frac{91}{6}$



(5) a) Talep fonk $\Rightarrow x = \sqrt{y/2}$

$2x^2 = y$

Arz fonk $\Rightarrow x = 2\sqrt{225-y}$

$y = 225 - \frac{x^2}{4}$

$2x^2 = 225 - \frac{x^2}{4}$

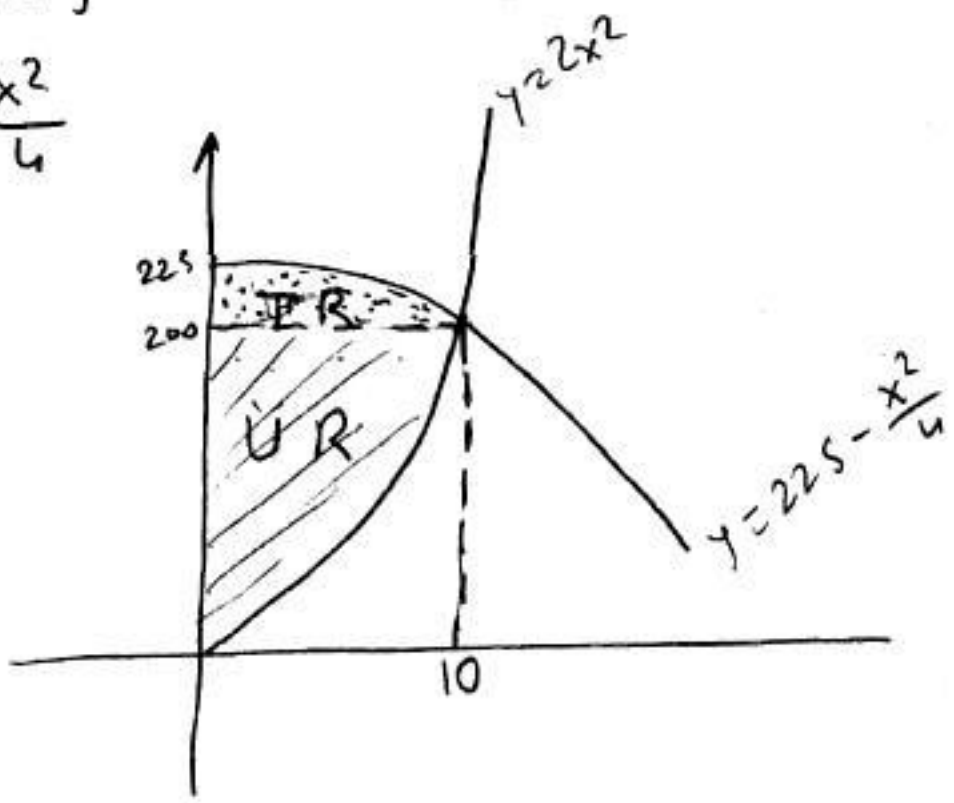
$2x^2 + \frac{x^2}{4} = 225$

$9x^2 = 900$

$x^2 = 100$

$\boxed{x_1 = 10}$

$x_2 = -10$



Tük. Rantı $= TR = \int_0^{10} \left(225 - \frac{x^2}{4}\right) dx - (10 \times 200)$

$= 225x - \frac{x^3}{12} \Big|_0^{10} - 2000$

$= \left(2250 - \frac{1000}{12}\right) - (0) - 2000$

$= 250 - \frac{250}{3} = \frac{500}{3}$.

Üretici Rantı $= (10 \times 200) - \int_0^{10} 2x^2 \, dx$

$= 2000 - \frac{2}{3} x^3 \Big|_0^{10} = 2000 - \left(\frac{2}{3} \cdot 1000\right) = \frac{4000}{3}$ //

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b) Marjinal Maliyet $= y' = 2x + 15$

i) Top. Maliyet $= y = \int (2x+15) dx$

$y = x^2 + 15x + C$

Sabit maliyet $= 80$ pb ise $y(0) = 80 \Rightarrow C = 80$

Top. Maliyet $= y = x^2 + 15x + 80$

ii) Gelir = satış fiyatı x miktar

$= 40 \cdot x$ marjinal Gelir $= (\text{Top Gel})' = 40$ //

Top. Gel. $= 40x$

iii) Kar = Gelir - Maliyet

$= 40x - (x^2 + 15x + 80)$

$= -x^2 + 25x - 80$

max kar $= (\text{kar})' = 0$ noktasıdır.

$(\text{kar})' = -2x + 25 = 0$

$x = 25/2$

iv) $y(10) = 330$ pb $\quad y(20) = 880$

Top. Gel. $(10) = 400$ pb $\quad$ Top Gel $(20) = 800$

Kar $(10) = 70$ pb $\quad$ Kar $(20) = -20$ pb